January 11, 2013

# Theoretical and Experimental Status of Inclusive Semileptonic Decays and Fits for $|V_{cb}|$



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We review recent experimental and theoretical developments in inclusive semileptonic $B \to X_c \ell \nu$ decays. In particular, we discuss the determination of $|V_{cb}|$ and of the heavy quark masses through fits based on the Operator Product Expansion.

PRESENTED AT

the $6^{th}$ International Workshop
on the CKM Unitarity Triangle (CKM2010)
University of Warwick, UK, 6–10 September 2010

Semileptonic $b \to c$ transitions play an important role in various aspects of heavy flavor physics. They allow for a determination of the CKM matrix element $V_{cb}$, which is a crucial input in all Unitarity Triangle analyses: for instance the $(\bar{\rho}, \bar{\eta})$ constraint determined by $\varepsilon_K$ is very sensitive to the precise value of the Wolfenstein parameter $A$, which is essentially determined by $|V_{cb}|$. Moreover, the bottom quark mass and the hadronic parameters extracted from fits to inclusive semileptonic and radiative moments are key inputs for the inclusive $|V_{ub}|$ determination, the normalization of rare $B$ decays like $B \to X_s\gamma$, and various other $B$ physics applications.

We will review the present theoretical and experimental status of inclusive $B \to X_c\ell\nu$ decays, with particular emphasis on recent developments, among which new calculations and measurements.

# 1 Theoretical framework

Our understanding of inclusive semileptonic $B$ decays rests on a simple idea: as all final states are summed over in inclusive decays, the final quark hadronizes with unit probability and the transition amplitude is sensitive only to the long-distance dynamics of the initial $B$ meson, which can indeed be factorized. An Operator Product Expansion (OPE) allows us to express the non-perturbative physics in terms of matrix elements of local operators of dimension $d \geq 5$, while the Wilson coefficients are perturbative [1, 2]. The leading term in this double expansion in $\alpha_s$ and $\Lambda_{\rm QCD}/m_b$ is given by the free $b$ quark decay, and the first corrections are $O(\alpha_s)$ and $O(\Lambda^2_{\rm QCD}/m_b^2)$. The relevant parameters are the heavy quark masses $m_b$ and $m_c$, the strong coupling $\alpha_s$, and the matrix elements of the local operators: $\mu_\pi^2$ and $\mu_G^2$ at $O(1/m_b^2)$, $\rho_D^3$ and $\rho_{LS}^3$ at $O(1/m_b^3)$, etc. Since the OPE is valid only for sufficiently inclusive measurements and away from perturbative singularities, the relevant quantities to be measured are global shape parameters (the first few moments of various kinematic distributions) and the total rate. The former give information on the masses and matrix elements, the latter on $|V_{cb}|$. The OPE parameters describe universal properties of the $B$ meson and of the quarks and are useful in many applications.

The main ingredients for an accurate analysis of the experimental data have been known for some time. Two implementations are currently employed, based on either the kinetic scheme [3, 4, 5] or the $1S$ scheme [6]. They both include terms through $O(\alpha_s^2\beta_0)$ [7] and $O(1/m_b^3)$ [8] but they use different perturbative schemes, include a somewhat different choice of experimental data under specific assumptions, and estimate the theoretical uncertainty in two distinct ways. Nevertheless, it is reassuring that, as we will show below, the two methods yield very close results for $|V_{cb}|$.

The complete two-loop perturbative corrections to the width and moments of the lepton energy and hadronic mass distributions have been recently computed [9, 10] by both numerical and analytic methods. In general, using $\alpha_s(m_b)$ in the on-shell scheme,

the non-BLM corrections amount to about $-20\%$ of the two-loop BLM corrections. In the kinetic scheme with cutoff $\mu = 1\text{GeV}$, the perturbative expansion of the total width is

$$\Gamma[\bar{B} \to X_c e \bar{\nu}] \propto 1 - 0.96\,\frac{\alpha_s(m_b)}{\pi} - 0.48\beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 + 0.82 \left(\frac{\alpha_s}{\pi}\right)^2 + O(\alpha_s^3) \approx 0.916 \quad (1)$$

Higher order BLM corrections to the width and moments are also known [3, 7]. The resummed BLM result is numerically very close to the NNLO one [3]. The residual perturbative error in the total width is therefore about 1%.

Since the numerical results of [10] are available for a variety of lepton energy cuts and values of $m_c/m_b$, it is now possible to implement them in a global fit. In the normalized leptonic moments the perturbative corrections cancel to large extent, independently of the scheme, as hard gluon emission is comparatively suppressed. This pattern of cancellations, crucial for a correct estimate of the theoretical error, is confirmed by the complete $O(\alpha_s^2)$ calculation, although the numerical precision of the available results is not always sufficient to improve the final accuracy. The actual implementation in the kinetic scheme is under way.

Another source of significant theoretical uncertainty are the $O(\alpha_s \Lambda_{QCD}^2/m_b^2)$ corrections to the width and to the moments. Only the $O(\alpha_s \mu_\pi^2/m_b^2)$ terms are known [11]. A complete calculation of these effects has been recently performed in the case of inclusive radiative decays [12], where the $O(\alpha_s)$ correction increase the coefficient of $\mu_G^2$ in the rate by almost 20%. The extension of this calculation to the semileptonic case is in progress. In view of the numerical importance of $O(1/m_b^3)$ corrections, if the 1% precision in the width is to be reached, the effects $O(\alpha_s/m_b^3)$ will also be necessary.

For what concerns the higher order power corrections, a thorough analysis of $O(1/m_b^4)$ and $O(1/m_Q^5)$ effects has just appeared [13]. The main problem is the proliferation of non-perturbative parameters: *e.g.* as many as nine new expectation values appear at $O(1/m_b^4)$. As they cannot be fitted from experiment, in Ref. [13] they are estimated in the ground state saturation approximation, reducing them to the known $O(1/m_b^{2,3})$ parameters. In this approximation the total $O(1/m_Q^{4,5})$ correction to the width is about $+1.3\%$. The $O(1/m_Q^5)$ effects are dominated by $O(1/m_b^3 m_c^2)$ Intrinsic Charm contributions, amounting to $+0.7\%$ [14]. The actual effect on $|V_{cb}|$ depends also on the corrections to the moments. The authors of [13] estimate that the overall effect on $|V_{cb}|$ is a 0.4% increase, consistent with our preliminary implementation of these effects in the kinetic global fit. While this sets the scale of higher order power corrections, it is yet unclear how much the result depends on the assumptions made on the expectation values.

The first two moments of the photon energy distribution in $B \to X_s \gamma$ are also routinely included in the semileptonic fit. They are sensitive to $m_b$ and $\mu_\pi^2$ in particular. However, the experimental lower cut on the photon energy introduces a sensitivity to the Fermi motion of the $b$-quark inside the $B$ meson and tend to disrupt the OPE.

One can still resum the higher-order terms into a non-local distribution function and since the lowest integer moments of this function are given in terms of the local OPE parameters, one can parameterize it assuming different functional forms [5]. Another serious problem is that only the leading operator contributing to inclusive radiative decays admits an OPE. Therefore in principle unknown $O(\alpha_s \Lambda/m_b)$ contributions should be expected [15] and radiative moments should be considered with care in the context of high precision analyses.

# 2 Measurements of moments

BaBar has recently published a study of the hadronic mass spectrum $m_X$ in inclusive decays $B \to X_c \ell \nu$ [16]. The main steps of this analysis, based on a data sample of 232 million $\Upsilon(4S) \to B\bar{B}$ events, are: First, the decay of one $B$ meson in the event is fully reconstructed in a hadronic mode ($B_{\mathrm{tag}}$) and the associated tracks and clusters are removed from the event. Such a $B_{\mathrm{tag}}$ candidate can be found in about 0.4% of the $\Upsilon(4S)$ events with a signal purity of about 80%. Then, the semileptonic decay of the second $B$ meson in the event ($B_{\mathrm{sig}}$) is selected by searching for an identified charged lepton (electron or muon) with momentum above 0.8 GeV/$c$. Finally, all remaining particles in the event are combined to reconstruct the hadronic $X$ system. The resolution in $m_X$ is improved by a kinematic fit taking into account 4-momentum conservation and the consistency of the missing mass with a zero mass neutrino.

Still, the observed $m_X$ spectrum is distorted by resolution and acceptance effects and cannot be used directly to obtain the hadronic mass moments. BaBar implements a linear correction to obtain the true moments from the reconstructed ones. Different corrections are applied depending on the $X$ system multiplicity, $E_{\mathrm{miss}} - cp_{\mathrm{miss}}$ and the lepton momentum. In this way, BaBar measures the moments of the hadronic mass spectrum up to $\langle m_X^6 \rangle$ for minimum lepton energies ranging between 0.8 and 1.9 GeV.

This study also updates the previous BaBar measurement of the lepton energy moments in $B \to X_c \ell \nu$ [17] using new branching fraction measurements for background decays and improving the evaluation of systematic uncertainties. Also, the first measurement of combined hadronic mass and energy moments $\langle n_X^k \rangle$ with $k = 2, 4, 6$ is presented, where the latter are defined as $n_X^2 = m_X^2 c^4 - 2\tilde{\Lambda} E_X + \tilde{\Lambda}^2$, with $m_X$ and $E_X$ the mass and the energy of the $X$ system and $\tilde{\Lambda}$ a constant fixed to 0.65 GeV.

BaBar interprets their data using the OPE in the kinetic scheme [3, 4, 5] and performs a simultaneous fit to 12 hadronic mass moments (or 12 combined mass-energy moments), 13 lepton energy moments (including partial branching fractions as 'zero order' moments), and 3 photon energy moments in $B \to X_s \gamma$ [18, 19]. The results are given in Table 1.

Also the Belle collaboration has obtained measurements of the lepton energy $E_\ell$ and the hadronic mass spectrum $m_X$ in $B \to X_c \ell \nu$ using 152 million $\Upsilon(4S) \to$

Table 1: Results of the OPE fits in the kinetic scheme to the BaBar data [16]. The first uncertainty quoted is experimental, the second theoretical.

| | Hadronic moments | Mass-energy moments |
|---|---|---|
| $\lvert V_{cb}\rvert$ $(10^{-3})$ | $42.05 \pm 0.45 \pm 0.70$ | $41.91 \pm 0.48 \pm 0.70$ |
| $m_b$ (GeV) | $4.549 \pm 0.031 \pm 0.038$ | $4.556 \pm 0.034 \pm 0.041$ |
| $\mathcal{B}(B \to X_c \ell\nu)$ (%) | $10.64 \pm 0.17 \pm 0.06$ | $10.64 \pm 0.17 \pm 0.06$ |
| $\chi^2$/ndf. | 10.9/28 | 8.2/28 |

Table 2: Results of the OPE fits in the kinetic and 1S schemes to the Belle data [23].

| | Kinetic scheme | 1S scheme |
|---|---|---|
| $\lvert V_{cb}\rvert$ $(10^{-3})$ | $41.58 \pm 0.90$ | $41.56 \pm 0.68$ |
| $\mathcal{B}(B \to X_c \ell\nu)$ (%) | $10.49 \pm 0.23$ | $10.60 \pm 0.28$ |
| $\chi^2$/ndf. | 4.7/18 | 7.3/18 |

$B\bar{B}$ events [20, 21]. The experimental method is similar to the BaBar analysis discussed previously, *i.e.*, one $B$ meson is fully reconstructed in a hadronic mode and an identified lepton is required to select semileptonic decays of the second $B$. In the Belle analyses acceptance and finite resolution effects in the $E_\ell$ and $m_X$ spectra are corrected by unfolding using the SVD algorithm [22]. Belle measures $\langle E_\ell^k \rangle$ for $k = 0, 1, 2, 3, 4$ and minimum lepton energies ranging between 0.4 and 2.0 GeV. Moments of the hadronic mass $\langle m_X^k \rangle$ are measured for $k = 2, 4$ and minimum lepton energies between 0.7 and 1.9 GeV.

To obtain $|V_{cb}|$, Belle fits 14 moments of the lepton energy spectrum, 7 hadronic mass moments and 4 moments of the photon energy spectrum in $B \to X_s\gamma$ [23] to OPE expressions derived in the kinetic [3, 4, 5] and 1S schemes [6]. Both theoretical frameworks are considered independently and yield very consistent results with the Belle data, Table 2.

# 3 Global HFAG fit

The Heavy Flavor Averaging Group (HFAG) has performed as global analysis of inclusive observables in $B \to X_c \ell\nu$ and $B \to X_s\gamma$ decays to determine $|V_{cb}|$, the $b$-quark mass $m_b$ and the higher order parameters in the OPE description of these decays. This analysis combines data from the BaBar, Belle, CLEO, CDF and DELPHI experiments.

Table 3: Experimental data used in the HFAG analysis of inclusive $B \to X_c \ell \nu$ and $B \to X_s \gamma$ decays. In the table, $\langle E_\ell^k \rangle$, $\langle m_X^k \rangle$ and $\langle E_\gamma^k \rangle$ refer to the moments of the lepton energy and hadronic mass spectrum in $B \to X_c \ell \nu$ and to the photon energy moments in $B \to X_s \gamma$, respectively. The index $k$ specifies the order of the moments used.

| | |
|---|---|
| BaBar | $\langle E_\ell^k \rangle$: $k = 0, 1, 2, 3$ [16, 17], $\langle m_X^k \rangle$: $k = 2, 4, 6$ [16], $\langle E_\gamma^k \rangle$: $k = 1, 2$ [18, 19] |
| Belle | $\langle E_\ell^k \rangle$: $k = 0, 1, 2, 3$ [20], $\langle m_X^k \rangle$: $k = 2, 4$ [21], $\langle E_\gamma^k \rangle$: $k = 1, 2$ [25] |
| CDF | $\langle m_X^k \rangle$: $k = 2, 4$ [26] |
| CLEO | $\langle m_X^k \rangle$: $k = 2, 4$ [27], $\langle E_\gamma^k \rangle$: $k = 1$ [28] |
| DELPHI | $\langle E_\ell^k \rangle$: $k = 1, 2, 3$ [29], $\langle m_X^k \rangle$: $k = 2, 4$ [29] |

Table 4: Results of the HFAG global fit in the kinetic scheme. The first error on $|V_{cb}|$ is the uncertainty from the global fit, the second is the error in the average $B$ lifetime and the third error is an additional theoretical uncertainty arising from the calculation of $|V_{cb}|$.

| Input | $\lvert V_{cb}\rvert$ $(10^{-3})$ | $m_b^{\rm kin}$ (GeV) | $\mu_\pi^2$ (GeV$^2$) | $\chi^2$/ndf. |
|---|---|---|---|---|
| all moments | $41.85 \pm 0.42 \pm 0.09 \pm 0.59$ | $4.591 \pm 0.031$ | $0.454 \pm 0.038$ | 29.7/59 |
| $X_c \ell \nu$ only | $41.68 \pm 0.44 \pm 0.09 \pm 0.58$ | $4.646 \pm 0.047$ | $0.439 \pm 0.042$ | 24.2/48 |

The global fit is done both with expressions derived in the kinetic [3, 4, 5] and 1S schemes [6]. In both cases 7 free parameters are determined in the fit. The only external input used in the analysis is the average $B^0$ and $B^+$ lifetime.

The data used in the global fit is listed in Table 3. In total 66 measurements – 29 from BaBar, 25 from Belle and 12 from other experiments – are used. Note that the analysis in the 1S scheme still uses the BaBar 2004 hadronic moment measurements [24].

The results of the global fit in the kinetic scheme are given in Table 4 and Fig. 1. The results of the 1S scheme analysis are shown in Table 5. In both cases, the results with all moments and with $B \to X_c \ell \nu$ moments only are quoted.

# 4 Discussion

We have seen that the fits discussed in the previous section determine $m_b$ quite precisely. How does this $m_b$ determination compare with alternative determina-

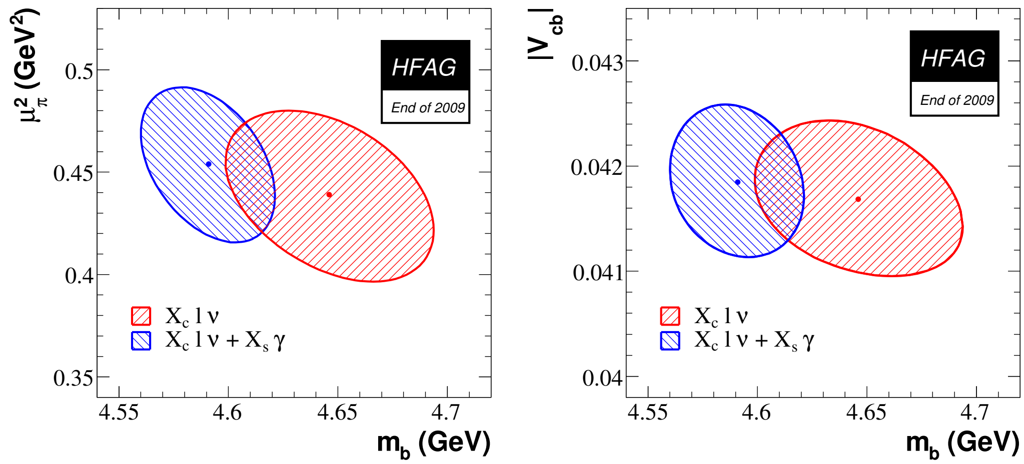


Figure 1: $\Delta\chi^2 = 1$ contours for the HFAG global fit in the kinetic mass scheme.

Table 5: Results of the HFAG global fit in the 1S scheme.

| Input | $\lvert V_{cb}\rvert$ $(10^{-3})$ | $m_b^{1S}$ (GeV) | $\lambda_1$ (GeV$^2$) | $\chi^2$/ndf. |
|---|---|---|---|---|
| all moments | $41.87 \pm 0.25$ | $4.685 \pm 0.029$ | $-0.373 \pm 0.052$ | 32.0/57 |
| $X_c\ell\nu$ only | $42.31 \pm 0.36$ | $4.619 \pm 0.047$ | $-0.427 \pm 0.057$ | 24.2/46 |

tions [30, 31, 32]? Semileptonic moments do not measure $m_b$ well. As illustrated in Fig. 2, they rather identify a strip in the $(m_c, m_b)$ plane along which the minimum is quite shallow, and $|V_{cb}|$ basically constant (straight lines). The global kinetic fit selects an $(m_c, m_b)$ region compatible with the loose PDG-2007 [33] bounds[1] as well as with the precise $e^+e^-$ sum-rules determinations [30, 31], of course after conversion to the kinetic scheme. This conversion is known to $O(\alpha_s^2)$ and entails a non-negligible error, of about 40 MeV for the conversion from $m_b^{\overline{\rm MS}}(m_b)$ to $m_b^{kin}(1\text{GeV})$ and about 10 MeV for that from $m_c^{\overline{\rm MS}}(3\text{GeV})$ to $m_c^{kin}(1\text{GeV})$.

It turns out that the semileptonic fit, and in particular its determination of the masses and the other OPE parameters, is very sensitive to various details. For instance, the assumptions on the correlations between theoretical errors for moments evaluated at different cuts have a clear impact on the OPE parameters, while the value of $|V_{cb}|$ remains quite stable. Such theoretical correlations are obviously hard to estimate. The present HFAG fit follows the procedure outlined in [34], assuming 100% correlation between moments calculated at different values of $E_{cut}$, the lower

[1] Later editions of PDG have stretched the uncertainties in an abnormal way.

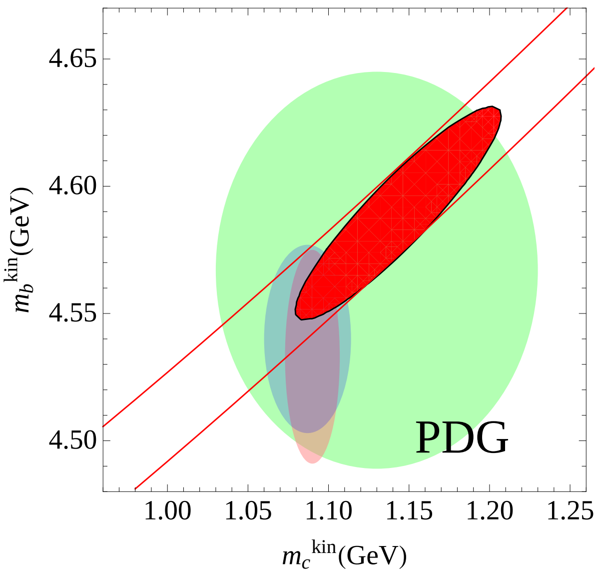


Figure 2: Different charm and bottom quark determinations in the kinetic mass scheme. The ellipses represent the PDG-2007 ranges (large green), a global semileptonic fit that differs slightly (see text) from the HFAG one (red), the Karlsruhe (pink) and Hoang *et al.* (blue) sum-rules determinations.

cut on the lepton energy. This very strong assumption distorts the fit, leading to high values of $m_{b,c}$, even outside the PDG range, and to underestimating the uncertainty of all non-perturbative parameters. On the opposite extreme, no correlation between close values of $E_{cut}$ is unreasonable. A more realistic approach, adopted in the fit shown in Fig. 2, consists in taking into account the $E_{cut}$ dependence and correlations of the known OPE calculation. It leads to slightly lower $m_{b,c}$ with larger errors. A detailed discussion will be presented elsewhere [35].

A related question concerns the role of radiative moments in the fits: as shown above they help fixing $m_b$. But the fit is almost identical if one replaces them with the loose bound $m_b^{\overline{\rm MS}}(m_b) = 4.20(7)$ GeV given by PDG in 2007. Indeed, the inclusion of external, well-founded constraints in the fit can be very useful: it decreases the errors and neutralizes the potential weight of theoretical correlations. As semileptonic decays do determine precisely a linear combination of $m_{b,c}$, a way to maximally exploit their potential consists in fitting directly $m_c^{\overline{\rm MS}}(3\text{GeV})$ instead of the kinetic charm mass (this is possible and avoids the scheme conversion error), and including in the fit one of the recent very precise $m_c$ determinations. As an illustration we have used $m_c^{\overline{\rm MS}}(3\text{GeV}) = 0.986(13)$ GeV by the Karlsruhe group [30], and obtained $m_b^{kin}(1\text{GeV}) = 4.535(21)$ GeV, which translates into $m_b^{\overline{\rm MS}}(m_b) = 4.165(45)$ GeV. This value for the bottom mass is perfectly consistent with the Karlsruhe group's own $m_b$

determination, $m_b^{\overline{\rm MS}}(m_b) = 4.163(16)$ GeV. The results of Refs. [31] and [32] are also consistent.

The kinetic scheme fitting routines are now undergoing a major upgrade, concerning the inclusion of higher order effects, the possibility to change the perturbative scheme, and the inclusion of additional constraints in the fit. The preliminary results we have just shown indicate that an uncertainty of about 20 MeV on $m_b$ can be reliably reached if an independent, precise determination of $m_c$ is employed. In view of this progress and of the calculations recently completed or under way, we believe that a 1% determination of $|V_{cb}|$ can be reached, although some work is still necessary.

## Acknowledgements

P.G. is grateful to N. Uraltsev for useful and informative discussions.

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